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J. Geogr. Inst. Cvijic. 66(2) (203–220)



Original scientific paper

UDC: 911.2:556.5(497.11)
DOI:10.2298/IJGH1602203U

AT-SITE HYDROLOGICAL DROUGHT ANALYSIS: CASE STUDY OF VELIKA MORAVA RIVER AT LJUBIČEVSKI MOST (SERBIA)

*Marko Urošev*¹, *Dragan Dolinaj*^{**}, *Dragoljub Štrbac*^{*}

^{*} Geographical Institute “Jovan Cvijić” SASA, Belgrade

^{**} Faculty of Sciences, University of Novi Sad, Novi Sad

Received: July 4, 2016; Reviewed: July 21, 2016; Accepted: August 3, 2016

Abstract: At-site frequency analysis of hydrological droughts is presented in this paper, in the example of the hydrological station Ljubičevski Most on the Velika Morava River, which represents the outlet of the entire Velika Morava basin, covering 42% of the Republic of Serbia. It is the first time that for the Velika Morava basin, and Serbia, theoretical distributions of deficit and duration of hydrological droughts are chosen according to best fit to empirical data, and not according to chosen in advance distributions, which has been the case until now. Also, for the first time in Serbia the method of L-moments was used for parameter estimation of distributions for extreme value modeling of hydrological drought characteristics. These improvements of existing method should contribute to better estimation of hydrological drought of large return period. The hydrological droughts were selected by threshold level method using daily data for the period 1960–2014, and their characteristics, deficits and durations of droughts were analyzed by method of partial duration series (peak over threshold). The results of calculations indicate that the best fit with the empirical data of deficit volumes has the model with binomial distribution of number of drought occurrences and Weibull distribution of exceedance magnitudes (B+W), and with drought durations model with binomial distribution of number of drought occurrences and exponential distribution of exceedance magnitudes (B+E). Based on the chosen distribution it is possible to calculate exceedance probabilities, i.e. return periods of deficit volumes and durations of largest observed droughts, like the 1993 drought, or to estimate 10-, 20-, 50- and 100-year droughts.

Key words: hydrological drought, method of threshold level, partial duration series, L-moments, the Velika Morava River

Introduction

People are exposed to a multitude of natural hazards around the world, such as earthquakes, volcanic eruptions, hurricanes, storms, tornadoes, floods and droughts. Hydrological extremes (floods and hydrological droughts) are natural hazards that are not limited to specific regions, but occur worldwide and, therefore, impact a very large number of people. Flooding events receive most attention, both in the news and in scientific literature, due to their fast, clearly

¹ Correspondence to: m.urosev@gi.sanu.ac.rs

visible, and dramatic consequences. Drought events develop slower and often unnoticed and have diverse and indirect consequences (Mishra & Singh, 2010). Hydrological droughts can, however, cover extensive areas and can last for months to years, with devastating impacts on the ecological system and many economic sectors, like: domestic and industrial water supply, irrigation, hydropower, navigation and recreation (Tallaksen & Van Lanen, 2004).

The first step in drought analysis is drought definition. Drought is a complex phenomenon so it can be defined in a number of different ways. There is no universal definition of drought (Lloyd-Hughes, 2014). The newest International glossary of hydrology (WMO & UNESCO, 2012) defines hydrological drought as “period of abnormally dry weather sufficiently prolonged to give rise to a shortage of water as evidenced by below normal streamflow and lake levels and/or the depletion of soil moisture and a lowering of groundwater levels”. In this paper the term hydrological drought is related to the hydrological droughts of surface waters, i.e. to the deficits of discharge in rivers. The threshold level method was used for selection of hydrological drought events on stations, so that hydrological drought is defined by two variables $X = f(D, T)$, where D — drought deficit, T — drought duration. Drought defined in this way provides more information for hydrological engineers than individual value of annual minimal discharge.

Study area

The analysis of hydrological droughts by threshold level method was done only for a few hydrological stations in Sebia: Sava–Sremska Mitrovica, Tisa–Senta, Danube–Bezdan and Danube–Bogojevo in the papers of Zelenhasić and Salvai (1987), Salvai, Srđević, & Zelenhasić (1990), Zelenhasić (2002). Radić and Mihailović (2006) presented different methods for deriving the constant and varying thresholds, as well as their influence on number, volume and duration of droughts for the Ljubičevski Most station on the Velika Morava River for the period 1951–2003, but without further frequency analysis, i.e. calculation of theoretical distribution for volumes and durations of droughts.

In this paper procedure for at-site frequency analysis of hydrological droughts is presented in the example of the hydrological station Ljubičevski Most on the Velika Morava river, which represents the outlet of the entire Velika Morava basin, so it could be assumed that calculated drought characteristics at this site represent the result of processes that take place in the basin, which are influenced by climatic and physical-geographic characteristics of the catchment. The basin area of the Velika Morava at Ljubičevski Most is 37,320 km², and in

the period 1960–2014 the mean annual water discharge was $230 \text{ m}^3/\text{s}$, annual maximum discharge of 1% exceedance probability ($Q_{\max 1\%}$) $2,344 \text{ m}^3/\text{s}$ and annual minimal discharge of 95% exceedance probability ($Q_{\min 95\%}$) $31.8 \text{ m}^3/\text{s}$.

The basin of the Velika Morava, the largest domicile river in Serbia, which covers 42% of the territory of Serbia, is chosen for hydrological drought analysis, because the Velika Morava represents the hydrological regime of rivers in Serbia, south of the Sava and the Danube (excluding the Drina and the Lim). Precipitation regime and air temperature influence the within year water flow variations. During winter in the large part of the Velika Morava basin snow cover interchangeably accumulates and melts, and in conditions of increased temperatures at this time of the year rainfall occurs, especially in the lower regions. In spring high waters develop due to rain and snowmelt in the mountains. In the summer–autumn period low flow occurs due to lack of precipitation and increased evapotranspiration. These processes influence the distribution of runoff within a year, so the rivers in the Velika Morava basin mostly belong to pluvio-nival type of water regime, with high water in spring (March and April), and low water in the summer–autumn period (August and September), so the analysis of hydrological drought is mostly related to the warm season (Figure 1). That is why the calculation was done for the calendar years (1 January–31 December).

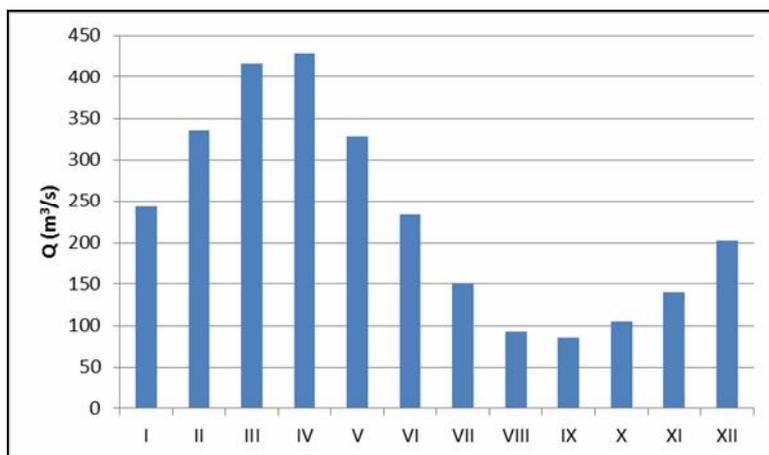


Figure 1. Mean monthly discharges of the Velika Morava at Ljubičevski Most for period 1960–2014

The period 1960–2014 was chosen because as long as possible data series is needed for further calculations of the distribution function of drought deficits and durations.

Methods

Selection of droughts by threshold level method

The threshold level (“truncation level”) method is discussed in details in Tallaksen, Madsen, and Clausen (1997) and Zelenhasić and Salvai (1987). The intensive use of this method is after year 1987, i.e. after the publication of the paper of Zelenhasić and Salvai who first used this method on daily discharge data. Further modifications of the method are related mainly to the way of the threshold selection and the grouping of interdependent droughts, as well as the removing of very small deficits. The manual on low-flow estimation and prediction of the World Meteorological Organization (WMO, 2008) also recommends the threshold level method for selection of hydrological droughts.

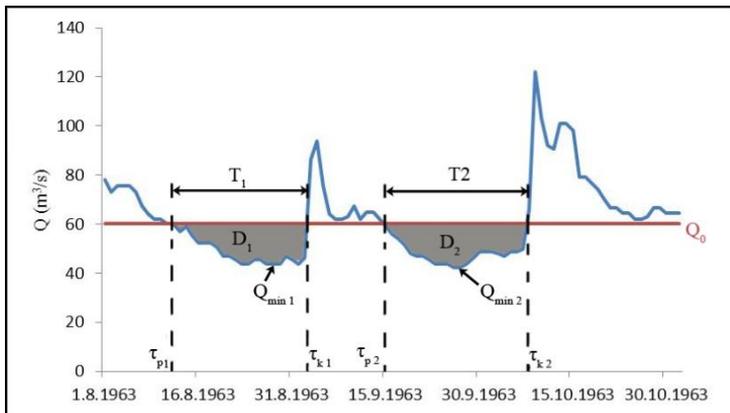


Figure 2. Defining hydrological drought characteristics

The example of selecting hydrological droughts by threshold method is presented in the Figure 2. In advance defined threshold value (Q_0) is applied on the observed daily hydrograph. Drought begins when discharge falls under threshold ($Q(t) < Q_0$) and drought ends when discharge returns above it ($Q(t) \geq Q_0$). This is how the time of beginning (τ_p) and ending of a drought (τ_k) is defined. The two most important characteristics of drought are duration (T) and deficit (D) (Figure 2). Duration (T) is the consecutive number of days when the discharge is under the threshold value. The deficit volume (D) (term severity or total deficit is also often used in literature) presents cumulative deficit of discharges ($D(t)$) for given drought duration and it is calculated with formulas (1) and (2).

$$D_i = \sum_{\tau_p}^{\tau_k} D(t) \quad (1)$$

where D_i is deficit (volume) of drought i (in m^3), and $D(t)$ is discharge deficit in time t and is equal:

$$D(t) = \begin{cases} Q_0 - Q(t), & Q(t) < Q_0 \\ 0, & Q(t) \geq Q_0 \end{cases} \quad (2)$$

The choice of threshold is very sensitive matter. The decision about height and type of threshold depends on purpose of drought study. Also, threshold could be in advance defined value, like specific inflow to the reservoir, or defined by user — water supply, navigation, hydropower, etc. It is possible to use some of the low flow characteristics, like percentage of mean flow, or percentile from the flow duration curve. Most often for threshold level the percentile from flow duration curve is used, for example Q_{95} , i.e. the discharge that is exceeded for 95% of time during observation period. For perennial rivers in mid latitudes, Q_{70} and Q_{90} are used as threshold levels. The threshold level that is very low could result in too many zero-drought years and the number of selected droughts is too small for frequency analysis. On the other hand, the threshold that is relatively high will select drought that lasts more than a year (multi-year droughts).

The threshold Q_{95} was first applied, because usually the goal of drought analysis is to calculate return periods of large droughts, i.e. the most extreme ones. This low threshold level has derived only 22 droughts at Ljubičevski Most on the Velika Morava, and in 36 out of 55 years (1960–2014) no droughts were recorded. This small number of droughts cannot give reliable estimation of distribution parameters that are necessary for frequency analysis of extreme droughts. The threshold defined by percentile Q_{90} (60.2 m^3/s) singled out greater number of droughts in relation to Q_{95} (52.1 m^3/s). For the same station 45 droughts (twice as much as Q_{95}) were singled out in the period 1960–2014, and droughts were recorded in 32 years, which enables sufficient number of data for reliable estimation of drought with little exceedance probabilities, i.e. large return periods. In the Figure 3. one can see how hydrological drought looks like on the station Ljubičevski Most (58.6 $\text{m}^3/\text{s} < Q_{90}$), recorded at the beginning of September 2015 at field survey. That is why the threshold Q_{90} was selected for drought defining, because generated time series are long enough, and the threshold is low enough to ensure that discharges included in the analysis belong to the lower extreme part of the hydrograph.

The use of daily data for defining droughts within year leads to two significant problems: dependence of droughts and minor droughts. During prolonged dry period discharge often exceeds the threshold for short period of time, so one drought is separated in a number of droughts. To avoid this problem, which can influence the extreme value modeling, the procedure for pooling these droughts should be introduced to gain independent time series of droughts. Tallaksen et al. (1997) described and analyzed three different procedures for pooling mutually dependent droughts: moving average (MA), sequent peak algorithm (SPA), the inter-event time and volume criterion (IC). They concluded that MA and SPA procedures give satisfactory results in pooling mutually dependent droughts and eliminating a number of minor droughts.



Figure 3. Hydrological drought on station Ljubičevski Most (Photo: M. Urošev)

MA procedure is applied on the time series before the selection of droughts. In this case the time series are smoothed and little peaks above threshold are removed. The use of 10 days averaging interval is recommended (Hisdal & Tallaksen, 2000). In this paper central moving average with 11 days interval (MA (11)) was used in order to preserve real dates when drought occurred. Although MA (11) filter removes great number of minor droughts and pools together mutually dependent droughts, but some of these events remain (Figure 4). For example, after the application of MA (11) filter on daily discharges of the Velika Morava at Ljubičevski Most, two such cases of dependent droughts and ten minor droughts remained.

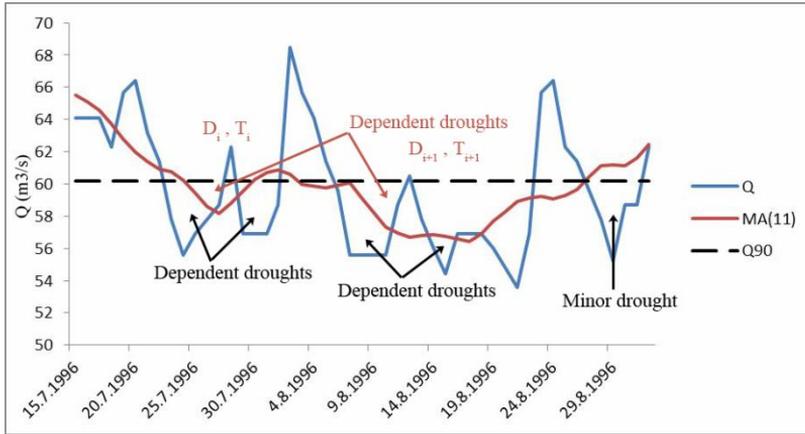


Figure 4 Influence of MA (11) filter on pooling mutually dependent droughts and removing minor droughts (Velika Morava — Ljubičevski Most, summer of 1996)

The solution for this problem implies the introduction of additional criterions. The first criterion is independence of droughts, i.e. the time between neighboring droughts needs to be greater than five days $t_c > 5$, because of applied filter MA (11) (five days before and five days after). If this criterion is not fulfilled neighboring droughts are pooled into one drought with the following characteristics:

$$\begin{cases} D'_i = D_i + D_{i+1} \\ T'_i = T_i + T_{i+1} \end{cases} \quad (3)$$

Analyzing droughts in the basins around the world, Fleig, Tallaksen, Hisdal, and Demuth (2006) concluded that the best combination for removing minor drought is minimal duration of drought $t_{\min} > 2$ days and minimal deficit $D_0 = 0.005 \times D_{\max}$, where D_{\max} is maximal observed deficit. These two criterions (t_{\min} and D_0) were used in this paper for removing minor droughts and also as location parameter for distributions of exceedance magnitudes of drought deficits and durations in PDS model.

Frequency analysis of deficits and durations of hydrological droughts

The two most commonly used methods for extreme value analysis are the annual maximum series (AMS) model and the *partial duration series* (PDS) model. The AMS consists of the largest event within each year, whereas the PDS contains all events above the threshold. Whether to use the AMS or the PDS model depends on the available data and the type of the analysis to be carried out. The

advantage of the AMS is easier definition of a series by selecting only the largest events within each year, while the advantage of the PDS model is the more consistent definition of the extreme value region, not taking only the largest annual value. In case of low threshold levels the occurrence of zero-drought years may significantly reduce the information content of the AMS. In the PDS, however, minor droughts may significantly distort the extreme value modeling, and a procedure for exclusion of minor droughts should be imposed (Hisdal & Tallaksen, 2000). Since heavy-tailed distributions, corresponding to negative shape parameters, are far the most common in hydrology, the PDS model generally is to be preferred for at-site quantile estimation (Madsen, Rasmussen, & Rosbjerg, 1997). That is why the PDS model was used in this paper for the frequency analysis of hydrological droughts.

The use of statistics and probability theory implies that hydrological series are homogeneous and independent. The test of autocorrelation of the first order and the test of squares of consecutive differences were used for testing independence (randomness). For testing the homogeneity of series several tests were used: parametric tests z-test and t-test for mean values and F-test for variance, and nonparametric test like Mann-Whitney (U-test) for distribution function. For testing the trend, parametric test of significance of correlation coefficient was used. For all tests level of significance was $\alpha = 0.05$. Definitions and formulas of the tests are well known and could be found in various references, for example Prohaska (2003).

The PDS consists of exceedances (“peaks”) which exceed some threshold value. Analysis of partial duration series is also known as peaks over threshold method (POT), which was developed by Todorović (Todorović, 1970; Todorović & Zelenhasić, 1970). The two biggest problems in the application of the method of partial duration series are the problem of exceedance independency and the selection of threshold. As it was already mentioned, time between two consecutive droughts larger than five days ($t_c > 5$) was used as a criterion of independence of droughts (exceedances), while minimum drought duration $t_{\min} > 2$ days and minimum deficit $D_0 = 0.005 \times D_{\max}$ were used as the thresholds for series of durations and deficits respectively. In the PDS method the term “base value” was used instead of the term threshold to make distinction with threshold (Q_{90}) which was used in the selection of droughts.

In random process drought deficits X_i in time interval $[0, t]$ which exceed some base value x_0 are analyzed (Figure 5). Then the values

$$Z_i = X_i - x_0, \quad i = 1, 2, \dots \quad (4)$$

represent exceedances or peaks above the threshold. The number of exceedances in the interval $[0, t]$ is random variable $\eta(t)$, which represents a discrete random process. The largest of all exceedances Z_i in interval $[0, t]$ is random process $\chi(t)$:

$$\chi(t) = \max \{Z_1, Z_2, \dots, Z_{\eta(t)}\}, \quad t \geq 0 \quad (5)$$

Assuming that exceedances are independent and identically distributed (test of homogeneity and independence of time series), the distribution of largest exceedance is equal to:

$$F(z; t) = P\{\chi(t) \leq z\} = P\{\eta(t) = 0\} + \sum_{n=1}^{\infty} [H(z)]^n \cdot P\{\eta(t) = n\} \quad (6)$$

where $H(z)$ is distribution function of exceedance, and $P\{\eta(t) = n\}$ distribution function of number of exceedances (Plavšić, 2006).

If interval $[0, t]$ is one year, then $\chi(t)$ is the largest annual exceedance above x_0 . Also, if the annual maximum deficit is marked with $X(t) = \chi(t) + x_0$, then the distribution of annual maximum is:

$$F(x) = P\{X(t) \leq x\} = P\{\chi(t) \leq x - x_0\} + \sum_{n=1}^{\infty} [H(x - x_0)]^n \cdot P\{\eta(t) = n\} \quad (7)$$

This distribution is defined only for $x > x_0$, i.e. for deficit larger than base value, while for deficits lower than base value it is not defined. When the distribution of annual maximum $F(x)$ is defined then the return period can be calculated:

$$T(x) = \frac{1}{P\{X > x\}} = \frac{1}{1 - F\{x\}}, \quad x > x_0 \quad (8)$$

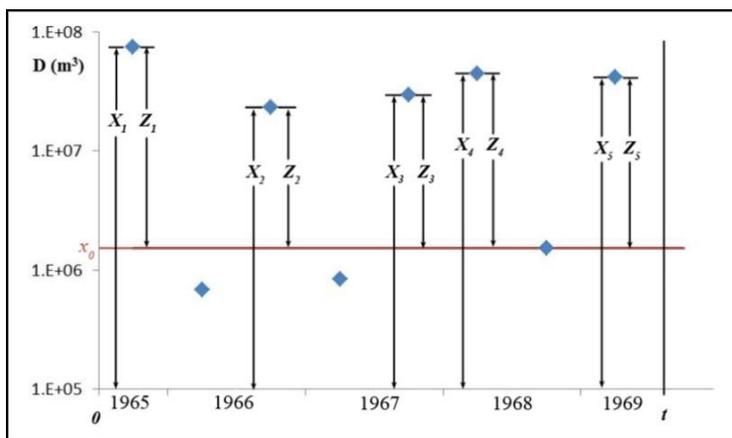


Figure 5. Deficits and exceedances over base value of the Velika Morava at Ljubičevski Most for the period 1965–1969

In the analysis of the number of exceedances usually Poisson distribution and binomial or negative binomial (depends on value of index of variance) distribution are used. The formulas of these distributions could be found, for example in Plavšić and Todorović (2015). Chi-square (χ^2) test was used for checking goodness of fit of empirical and theoretical distribution of number of drought occurrences.

The selection of distribution function of exceedances has significant influence on the estimation of extreme events (droughts), because there are significant differences between calculated quantiles in extreme parts of different distribution curves (tails). General recommendation for theoretical distributions of low flow and droughts is lower limits equal or greater than zero and not more than three parameters. The more parameters distribution has, better the fit to empirical data is, but the reliability of parameter estimation is lower. Most commonly exponential, Weibull and generalized Pareto distributions are used for exceedance magnitudes. Also only for models with these three distributions of exceedance magnitudes it is possible to find explicit analytical solution for quantiles $x(F)$ in the analysis of annual maximum exceedance. The formulas for distributions of exceedance magnitudes, as well as the formulas for the estimation of parameters by method of L-moments, used in this paper could be found, for example in Hosking and Wallis (1997).

The three main methods for parameter estimation are: method of moments, method of L-moments and method of maximum likelihood. Hosking (1990) defined L-moments as linear (hence the prefix “L”) combinations of ranked

observations. The main advantage of L-moments over conventional moments is that they suffer less from the effects of sampling variability, because they do not involve squaring or cubing the observations as the conventional moments do. As a result, the estimations of dimensionless L-Cv and L-Cs are unbiased and have very nearly normal distributions. In a wide range of hydrological applications L-moments provide simple and reasonably efficient estimators of characteristics of hydrological data and distribution parameters, especially from small samples (Stedinger, Vogel, & Foufoula-Georgiou, 1993). Kolmogorov-Smirnov (D_{max}) and Cramer-von-Mises (Nw^2) goodness of fit tests were used for determination of distribution of exceedance magnitudes.

The distribution of annual maximum $F(x)$ was given earlier in the formula (7). From this general expression (7), the formulas for different partial duration series models can be derived. The models assume different combinations of distribution of number of exceedances $P\{\eta = n\}$ and distribution of exceedances $H(z)$. The distribution of annual maximum deficits and the durations of droughts are chosen according to the results of goodness of fit tests and probability plot.

Results

Hydrological droughts are selected by the threshold level method, i.e. the series of drought deficits and durations are formed and ready for the statistical analysis (Table 1). If we compare the first five largest droughts according to deficit with durations relative to them, we can see there are some deviations. For example, the maximum deficit was observed in the year 1993 ($305 \cdot 10^6 \text{ m}^3$), and according to duration (148 days) this drought is ranked second behind the year 1990 (151 days). However, in general it can be said that there is a strong relationship between the deficits and durations of droughts. For the station of Ljubičevski Most, the coefficient of determination of linear regression R^2 is 0.80.

Table 1. Hydrological droughts of the Velika Morava river at Ljubičevski Most ($Q_{90} = 60.2 \text{ m}^3/\text{s}$, $t_c > 5$, $t_{\min} > 2$, $D_0 = 1,52 \cdot 10^6 \text{ m}^3$)

Date of drought beginning	Date of drought end	D (10^6 m^3)	T (days)	Date of drought beginning	Date of drought end	D (10^6 m^3)	T (days)
28.08.1960.	25.09.1960.	11.8	29	15.09.1992.	14.10.1992.	17.6	30
03.10.1960.	13.10.1960.	3.4	11	23.06.1993.	17.11.1993.	304.8	148
03.08.1961.	10.11.1961.	176.1	100	01.08.1994.	18.10.1994.	127.6	79
10.08.1962.	17.10.1962.	76.9	69	24.10.1994.	04.11.1994.	1.8	11
12.08.1963.	02.09.1963.	17.2	22	25.07.1996.	26.08.1996.	4.5	29
14.09.1963.	04.10.1963.	19.6	21	23.09.1997.	04.10.1997.	3.5	12
02.08.1965.	19.11.1965.	73.8	110	29.07.1998.	11.09.1998.	46.0	45
15.09.1966.	28.10.1966.	23.2	44	02.10.2000.	16.10.2000.	2.7	15
27.09.1967.	06.12.1967.	29.6	71	24.10.2000.	09.11.2000.	2.9	17
06.07.1968.	16.08.1968.	44.8	42	08.12.2000.	25.12.2000.	3.1	18
02.10.1969.	02.12.1969.	41.8	62	15.08.2003.	12.09.2003.	9.1	29
19.08.1971.	13.09.1971.	15.4	26	23.09.2003.	05.10.2003.	4.0	13
22.06.1972.	13.07.1972.	25.5	22	20.08.2007.	12.09.2007.	16.5	24
12.08.1972.	18.08.1972.	3.4	7	22.09.2007.	08.10.2007.	6.5	17
16.09.1974.	26.09.1974.	3.2	11	24.08.2008.	15.09.2008.	9.3	23
06.09.1977.	16.09.1977.	2.2	11	13.11.2008.	21.11.2008.	2.0	9
24.09.1982.	04.10.1982.	3.4	11	22.08.2011.	19.10.2011.	39.1	59
30.05.1983.	09.06.1983.	1.8	11	29.10.2011.	16.11.2011.	6.2	19
18.07.1985.	25.08.1985.	44.7	39	05.08.2012.	07.11.2012.	88.4	95
15.09.1985.	02.11.1985.	47.7	49	24.11.2012.	06.12.2012.	6.8	13
05.09.1987.	12.10.1987.	17.2	38	29.07.2013.	21.09.2013.	20.4	55
07.08.1988.	15.09.1988.	10.1	40	29.10.2013.	15.11.2013.	18.1	18
08.07.1990.	05.12.1990.	190.6	151				

The results of the test have shown that the series of deficit and durations of drought are homogeneous and independent, i.e. they fulfill the conditions for further statistical analysis. The chosen threshold Q_{90} singled out 45 droughts (Table 1) in the period 1960-2014. The mean annual number of droughts is $\bar{\eta} = 0.82$, variance $S_{\eta}^2 = 0.67$ and an index of variance $I = 0.82$. Since $I < 1$, besides Poisson distribution, calculations were made for binomial distribution. In the studied 55 year period there were 23 years without drought (probability of occurrence $P = 0.418$), 20 years with one ($P = 0.364$), 11 years with two ($P = 0.200$) and one year with three droughts ($P = 0.018$). According to the results of χ^2 -test for the level of significance $\alpha = 0.05$ both theoretical distributions fit empirical data, but according to p -value better fit has Poisson distribution (0.42) in relation to binomial (0.29).

The parameters and values of cumulated distribution functions of deficit and duration exceedances were determined based on the calculated L-moments of

the sample (Table 2). According to the results of goodness of fit tests (Kolmogorov-Smirnov and Cramer-von-Mises), Weibull and generalized Pareto distributions fit observed deficits, while Weibull distribution has better fit. All distributions are in accordance with the empirical values of drought durations, while the differences in values are much less than for deficits. Weibull distribution has the lowest values of test statistics, so this distribution was also chosen for drought durations.

Table 2. L-moments of exceedance magnitudes of deficits (D) and drought durations (T) on the Velika Morava at Ljubičevski Most

Variable	λ_1	λ_2	λ_3	λ_4	τ_2^*	τ_3^*	τ_4^*
D (10^6 m^3)	34.6	24.4	14.4	8.9	0.71	0.59	0.36
T (days)	37.4	17.8	7.1	3.3	0.47	0.40	0.18

* τ_2, τ_3, τ_4 are dimensionless L-moments

Distributions of annual maximum deficit and duration of droughts are presented in the Figures 6 and 7 respectively. In order to easy determine quantiles and better check visually, distribution functions of annual maximums are shown on Gumbel probability plot. According to the results of goodness of fit tests, the best fit has a combination of binomial distribution for the number of drought occurrences and Weibull distribution for exceedance magnitudes, i.e. the model B+W.

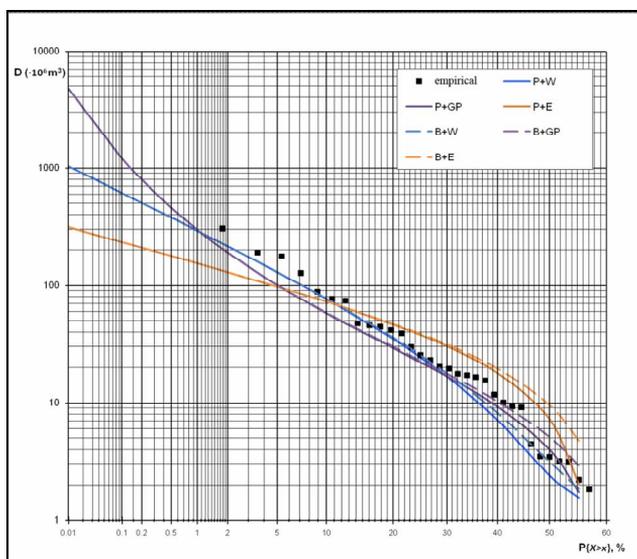


Figure 6. Exceedance probability of annual maximum drought deficits on the Velika Morava at Ljubičevski Most

Small probabilities of exceedance, i.e. large return periods, are always important in extreme value analysis, so after the goodness of fit tests there is a need to check visually on probability plot which distribution approximates best extreme events, the ones with exceedance probabilities $P\{X > x\}$ less than 0.1 or 10%. Since the model B+W describes best the upper part of empirical distribution on the probability plot (Figure 6), we accept this combination of distributions as design model for annual maximum deficits.

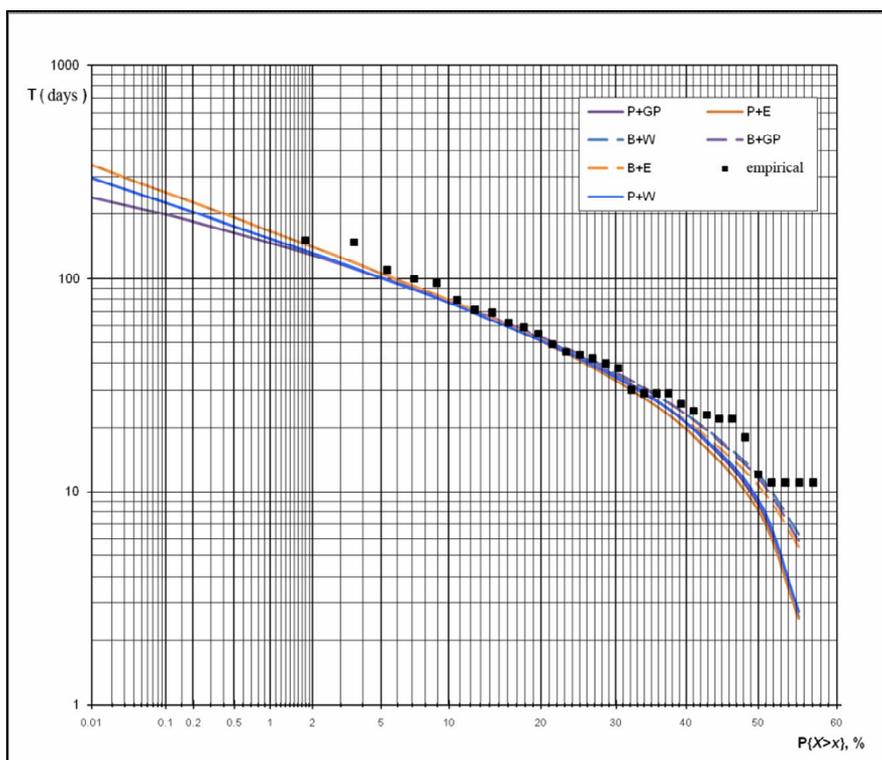


Figure 7. Exceedance probability of annual maximum drought durations on Velika Morava at Ljubičevski Most

The results of goodness of fit tests show that B+W model is the best for annual maximum drought durations. Kolmogorov-Smirnov and Cramer-von-Mises tests evaluate the whole range of observed values, with large and also small exceedance probabilities. However, if we look at the upper part of distribution curves on the probability plot ($P\{X > x\} < 10\%$), which is in the focus of this research, we can see that the curve of exponential distribution is closest to the empirical values, and considering that model B+E has three parameters, it is accepted as design model for annual maximum drought durations (Figure 7).

It can be seen on both probability plots that for annual maximum distribution the distribution of exceedance magnitudes (continual distributions) is more important than number of occurrences (discrete distributions). That is why the models that have the same distribution of exceedance magnitudes have differences only in the lower part of the curves, i.e. for small return periods (Figures 6 and 7). For example, accepted distribution B+W for annual maximum deficits has no differences from distribution P+W for probabilities $P < 30\%$, i.e. for determination of deficits of small exceedance probabilities it is not important which model is used, B+W or P+W. It can be concluded that for same distributions of exceedance magnitudes binomial distribution provides bigger quantile values (drought deficit or drought duration) in relation to Poisson distribution for bigger exceedance probabilities around $P > 30\%$ (Figures 6 and 7), while negative binomial for same probabilities gives lower values of quantiles. This insight on model attributes can help with choice of regional distribution, because determination of parameter of Poisson distribution is much easier than for binomial or negative binomial distribution.

What can a researcher, an engineer or any other person get from the results presented in this paper? Firstly, the deficits and durations of drought of any return period for the station Ljubičevski Most on the Velika Morava Rriver can be calculated. For example, the 10-year, 50-year and 100-year droughts are most commonly of interest in the frequency analysis (Table 3).

Table 3. Deficits and drought durations of characteristic return periods on Velika Morava at Ljubicevski Most

Variable	Return period (years)						
	10	20	50	100	200	500	1000
Deficit ($\cdot 10^6 \text{ m}^3$)	77	130	216	293	380	508	617
Duration (days)	79	106	141	167	193	227	253

All that one should do is to read out the value of quantile for design model, i.e. the value of deficit for wanted return period, from probability plot of annual maximum deficit (Figure 6). Also, it is possible to analytically calculate drought of wanted return period using formula for respective distribution, whose parameters can be calculated using L-moments provided in the Table 2. Besides, the opposite procedure is possible, i.e. for any observed extreme drought in future, based on the deficit and duration, using probability curves or formulas, the return period can be determined. For example, the maximum deficit in the period 1960–2014 was observed in the year 1993 – $305 \cdot 10^6 \text{ m}^3$, which according to B+W model corresponds to the return period of 110 years, while for the drought duration of 148 days and B+E model, it corresponds to the drought of 61-year return period.

Conclusion

Low flow and droughts affect many aspects of the environment and society, and future increase in the demand for water will be the most critical in the periods of severe droughts. In terms of hydrological research there is a need to improve our ability to predict the onset, duration and severity of a drought, thus providing a better basis for the design of drought facilities. Drought studies and frequency analysis of historical data provide in this respect an important contribution to improve knowledge of the status and dynamics of water resources.

The advantage of the analysis of low flows, that is, hydrological droughts with two variables (deficit and duration) in relation to common analysis with one value, minimal annual discharge, is presented in this paper. The procedure applied in the paper could also be applied in the frequency analysis of high water, i.e. floods; only instead of the volumes below the threshold level, the flood volumes above Q_{10} or Q_5 are selected. The choice of the threshold Q_{90} influenced the choice of the method of partial duration series for the frequency analysis of drought characteristics. Since the low threshold value and relatively small number of selected droughts, the PDS method performed better than the AMS method for frequency analysis of drought deficits and durations. L-moments, which provide more reliable estimation of parameters in relation to ordinary moments, were used for parameter estimation of theoretical distributions. This is the first time in Serbia, according to author's knowledge, that L-moments were used in the analysis of extreme hydrological events, defined by the method of threshold level

At-site statistical analysis of hydrological droughts is presented in the paper. Distributions of the number of drought occurrences were tested with χ^2 test, and exceedance magnitudes and annual maximums (the largest exceedances) of deficits and durations were tested by Kolmogorov-Smirnov and Cramer-von-Mises tests and according to the results, proper distributions were adopted. In the previous papers which analyzed hydrological droughts with partial duration series, the distribution was defined in advance, and not on the basis of goodness of fit test and probability plot check. The results of calculations enable the estimation of the quantile of any return period (for example, 100-year deficit) and vice versa, which can be useful when planning water management measures and designing different hydrotechnical facilities.

Acknowledgements

This paper presents the result of work on the project "Geography of Serbia" (project No 47007) financed by the Ministry of Education, Science and Technological Development of the Republic of Serbia.

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